Nonneutrality of Money in Dispersion: Hume Revisited

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Abstract

For a class of standard and widely-used preferences, a one-shot money injection in a standard matching model can induce a significant and persistent output response by dispersing the distribution of wealth. Decentralized trade matters for both persistence and significance. In the presence of government bonds the injection has a liquidity effect and the inflation rate following the injection may be below the steady-state rate level.

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1 Introduction

First published in 1752, *Of Money* articulates Hume's view of nonneutrality of money that has been influential for centuries:

> At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another, till the whole at last reaches a just proportion with the new quantity of specie which is in the kingdom....When any quantity of money is imported into a nation, it is not at first dispersed into many hands; but is confined to the coffers of a few persons, who immediately seek to employ it to advantage....It is easy to trace the money in its progress through the whole commonwealth; where we shall find, that it must first quicken the diligence of every individual, before it encrease the price of labour. [Hume [20, p 172]]

In this heavily-cited passage,[1] Hume seemed to relate a stimulating effect of a money injection to (a) a limited participation in the market from which money is injected and (b) a dispersion (i.e., diffusion) process in the market from which injected money gradually reaches all people in the economy. But is there any mechanism by which a limited participation and a dispersion process can make an injection stimulating? Here we explore such a mechanism against the familiar matching model of Trejos and Wright [35] and Shi [33] with general individual money holdings, a model that accommodates a limited participation by nondegenerate wealth distributions and a gradual dispersion process by decentralized trade.[2] We use a class of standard and widely-used preferences for quantitative exercises and concentrate on one-shot money injections.

Our parameterized model has two salient features. First, aggregate output would increase in a steady state if people's incentives to trade were not changed but the distribution of wealth were more dispersed. The main force behind is simple and intuitive: a reduction in a poor seller's wealth results in a much larger increment in production than the reduction in a rich seller's. Secondly, people's incentives along a transitional path to the steady state are very close to their incentives in the steady state. These two features imply that if a redistributional shock disperses (stretches) the steady-state distribution of money but maintains the quantity of money, there is an immediate significant output response. An obvious role of decentralize trade is to add persistence by slowing down the dispersion (diffusion) of redistributed money.

[1] For its influence on contemporary monetary economics, see, e.g., Friedman [15], Lucas [28], and Wallace [37].

[2] The canonical form of the model, one with divisible money and with no upper bound on the individual holdings, has a central role in the New Monetarism literature (see Williamson and Wright [42]) in that much of the literature is built on its tractable versions, e.g., Trejos and Wright [35] and Shi [33] with indivisible money and a unit upper bound on the individual money holdings, and Lagos and Wright [24] and Shi [34] with different new ingredients.

There is a more critical role of decentralized trade. Suppose that a competitive market substitutes for decentralized trade as in a Bewley model. If there is no change in the price level, then the wealth redistribution may still have the same output effect as above; but for the market to clear, nominal prices must fall below the steady-state level in the transitional path, which, in turn, dilutes sellers' incentives to produce and dampens the output response. In other words, a suitable wealth redistribution is able to exploit the steady-state incentives to trade—in particular, poorer sellers' much stronger incentives to produce—because these incentives are preserved by decentralized trade in the transitional path. Apparently, such a redistribution can be done by a regressive money injection.

It is certainly not new that a money injection is nonneutral when it redistributes wealth (see Friedman [14]) and redistribution effects have drawn a fair amount of attention.[3] Our contribution is to reveal a mechanism by which some wealth redistribution may induce significant and persistent output responses. Such output responses, as is well known, are hard to come by in a large class of models absent of imposed nominal rigidity (see, e.g., Chari, Kehoe, and McGrattan [8]); while one may therefore appeal to nominal rigidity, there is always criticism of the assumption that people cannot change prices when they want.[4] In this context, relevance of our contribution is to show that price flexibility can be consistent with the output-response pattern in concern and, as noted below, with observed nominal rigidity.

To discipline our exercises, we endogenize regressiveness of each money injection as we endogenize a limited participation to the injection. With a unitary CRRA coefficient and a unitary Frisch elasticity of labor supply, a 1% accumulative increase in the money stock can induce a more than 4% accumulative increase in output over 20 quarters. There is a Phillips curve in that the output and price responses are proportional to the increase in the money stock.

When the model includes nominal government bonds (issued before pairwise meetings), inflation in a steady state is driven by interest payments to these bonds. In the steady state people carry only a small portion of nominal wealth in money into pairwise meetings; as implied by the second salient feature of the model, most of injected money must go to the bond market so there is a liquidity effect.[5] The output response remains significant and persistent. The inflation rate may first drop below the steady-state level, a phenomena analogous to what is referred to as the price puzzle in some VAR studies (see, e.g., Christiano, Eichenbaum, and Evans [9]). The key

[3]Some familiar studies come from the limited-participation literature. Redistribution effects are explored by the early and some late contributions in this literature; see, e.g., Grossman and Weiss [17], Rotemberg [32], Alvarez, Atkeson, and Kehoe [2] and Williamson [40, 41].

[4]A defense to nominal rigidity is that there may be some costs to changing prices, e.g., menu costs (Mankiw [29]) or nominal contracts. See Head, Liu, Menzio, and Wright [19] for a comprehensive review of the literature.

[5]For comparison, a liquidity effect arises in models of limited participation (see, e.g., Grossman and Weiss [17], Rotemberg [32], and Lucas [27]) because some money in circulation cannot reach the bond market when money is injected.

is that the injection increases the value of money at the present period by reducing interest payments and, hence, the money growth in the next period.

We spell out the basic model, parameterization, and the procedure for quantitative exercises in section 2. Section 3 demonstrates the two salient features of the model and the critical role of decentralized trade. The one-shot regressive injection is introduced in section 4. The model with bonds is presented in section 5. In section 6, we offer some discussion of our model, findings, and future works.

2 The basic model

The model is the one formulated by Trejos and Wright [35] and Shi [33] with general individual money holdings. Time is discrete, dated as $t \geq 0$. There is a unit mass of infinitely lived agents. At each period, each agent has the equal chance to be a buyer or a seller. Each buyer is randomly matched with a seller. In each pairwise meeting, the seller can produce a consumption good only consumed by the buyer.[6] The good is divisible and perishes at the end of the period. By exerting l units of the labor input, each seller can produce $y = Al$ units of goods, where $A > 0$. If the seller exerts l units of the labor input, his disutility is

$$c(l) = l^{1+1/\eta}/(1+1/\eta),\ \eta > 0. \tag{1}$$

The buyer is also endowed $\omega > 0$ amount of goods; if he receives y units of goods from the seller, his period utility is

$$u(y) = (y+\omega)^{1-\sigma}/(1-\sigma),\ \delta > 0. \tag{2}$$

Each agent maximizes his expected utility with a discount factor $\beta \in (0,1)$. There exists a durable and intrinsically useless object, called money. Money is indivisible and its smallest unit is Δ; the initial average holdings of money is M; and there is a finite but arbitrarily large upper bound B on the individual money holdings.[7] The initial distribution of money, denoted π_0, is public information.

In each pairwise meeting, each agent can observe his meeting partner's money holdings, but not past trading histories, which rules out credits between the two agents. Following a convention introduced by Berentsen, Molico, and Wright [4] into matching models with indivisible money, we allow stochastic trade so that a meeting

[6] One may add one dimension into the model by assuming that there is a probability for the meeting to be a non-coincidence meeting. The probability matters for the equilibrium level of output but has little effect on the output response to a shock.

[7] If $(\Delta, B) = (0, \infty)$, that is, money is divisible and there is no upper bound on money holdings, then the choice of M only affects prices. But numerical analysis still needs some $B' < \infty$ to approximate $B = \infty$; and the grid method needs some positive Δ' to approximate $\Delta = 0$. In this regard, the present model saves us from a few layers of approximation. Analytically, Zhu [45] gives a sense that $\Delta' > 0$ approximates $\Delta = 0$ when $B < \infty$ and M are fixed. In our numerical analysis below, computed equilibria seem to converge as B increases when Δ and M are fixed.

outcome is a lottery on the feasible transfers of goods and money.[8] Because $u(.)$ is strictly concave and $c(.)$ is strictly convex, it is not optimal for agents to choose a lottery which randomizes on the transfer of goods. So it is without loss of generality to represent a generic meeting outcome by a pair (y, μ), where $y \geq 0$ is the transfer of goods and μ is a probability measure on $\left\{0, \ldots, \min\left(m^b, B - m^s\right)\right\}$, meaning that the probability for the buyer to transfer d units of money to the seller is $\mu(d)$.

To define equilibrium, let v_t be the value function and π_t be the distribution of money at the start of period t; that is, for $m \in B_\Delta \equiv \{0, \Delta, \ldots, B\}$, $v_t(m)$ is the expected discounted utility for an agent holding m units of money and $\pi_t(m)$ is the proportion of agents holding m at the start of period t. Consider a seller with $m^s \in B_\Delta$ meets a buyer with $m^b \in B_\Delta$ at period t. To preserve concavity of value functions, we follow some recent treatment in matching models of money to let the outcome in the meeting be determined by the weighted egalitarian solution of Kalai [21];[9] that is, the equilibrium meeting outcome is

$$\left(y\left(m^b, m^s, v_{t+1}\right), \mu\left(m^b, m^s, v_{t+1}\right)\right) = \arg\max_{y,\mu} S^b\left(y, \mu, m^b, m^s, v_{t+1}\right) \tag{3}$$

subject to

$$\theta S^s\left(y, \mu, m^b, m^s, v_{t+1}\right) = (1-\theta) S^b\left(y, \mu, m^b, m^s, v_{t+1}\right), \tag{4}$$

where

$$S^b\left(y, \mu, m^b, m^s, v_{t+1}\right) = u(y) - u(0) + \beta \sum_d \mu(d) \left[v_{t+1}\left(m^b - d\right) - v_{t+1}\left(m^b\right)\right] \tag{5}$$

is the buyer's surplus from trading (y, μ),

$$S^s\left(y, \mu, m^b, m^s, v_{t+1}\right) = -c(y/A) + \beta \sum_d \mu(d) \left[v_{t+1}(m^s + d) - v_{t+1}(m^s)\right] \tag{6}$$

is the seller's surplus, and θ is the buyer's share of surplus. Let $f^b\left(m^b, m^s, v_{t+1}\right)$ and $f^s\left(m^b, m^s, v_{t+1}\right)$, respectively, denote the buyer's surplus and the seller's surplus implied by the equilibrium meeting outcome. Then given (v_{t+1}, π_t), v_t satisfies

$$v_t(m) = \beta v_{t+1}(m) + 0.5 \sum_{m'} \pi_t(m') \left[f^b(m, m', v_{t+1}) + f^s(m', m, v_{t+1})\right]; \tag{7}$$

π_{t+1} satisfies

$$\pi_{t+1}(m) = \sum_{m'} \lambda(m', m, v_{t+1}) \pi_t(m'), \tag{8}$$

where $\lambda(m', m, v_{t+1})$ is the proportion of agents with m' units of money leaving with m after pairwise meetings in period t implied by money transfer lotteries $\{\mu(\cdot; m^b, m^s, v_{t+1}) : (m^b, m^s) \in B_\Delta \times B_\Delta\}$ and is explicitly described in the appendix.

[8] Provided that the function v_{t+1} is concave, i.e., $2v_{t+1}(m) \geq v_{t+1}(m+\Delta) + v_{t+1}(m-\Delta)$ for $0 < m < B - \Delta$, lotteries in the meeting convexify the set of surpluses from trade, making the bargaining problem in (3) well defined. Convexification aside, lotteries ensure that neutrality of money may be well approximated by a relatively small M/Δ.

[9] See, e.g., Aruoba, Rocheteau, and Waller [3], Lester, Postlewaite, and Wright [25] and Venkateswaran and Wright [36]. In Trejos and Wright [35] and Shi [33], the trade in the meeting is determined by the generalized Nash bargaining solution, treated as the limit of equilibria from an alternating-offer game in the meeting. This Nash solution does not guarantee concavity of value functions in equilibrium; in numerical analysis it certainly does not preserve concavity in iterations. As is well known, concavity is preserved under Kalai bargaining.

Definition 1 *Given π_0, a sequence $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ is an equilibrium in the economy if it satisfies* (3)-(8)*. An equilibrium is a monetary equilibrium if $v_t(B) > 0$ for some t. A pair (v, π) is a steady state if $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ with $v_t = v$ and $\pi_t = \pi$ for all t is an equilibrium.*

Given parameter values, we apply a two-step procedure of numerical analysis to study the output and price responses to an unanticipated shock in sections 3 and 4.

Step 1. We compute a steady state (v, π) such that v is strictly increasing and concave (for concavity see footnote 8). When θ is sufficiently close to one, we can adapt the proof in Zhu [44] to show existence of such a steady state if ω is sufficiently small; we cannot extend that proof for a general θ. But existence holds if we perturb the model so that money yields arbitrarily small direct utility. When perturbation goes to zero, a limit of steady states in perturbed models is a steady state in the (original) model and it is a desired steady state if the limit value function is not a zero function. We test whether a computed steady state (v, π) is an object that truly exists by testing whether it is approximated by steady states in perturbed models.

After we obtain numerical values of (v, π), we check its local stability as follows. First, we obtain a dynamic system $(v_{t+1}, \pi_{t+1}) = \Phi(v_t, \pi_t)$ in a neighborhood of (v, π) from the definition of equilibrium. For this system, we need that v_{t+1} is solvable from the equilibrium condition (7), treating (v_t, π_t) as parameters; this is up to checking whether the relevant Jacobian is of full rank. Next we compute eigenvalues of the Jacobian of $\Phi(\cdot, \cdot)$ evaluated at (v, π). Based on the number of eigenvalues inside the unit circle, we are able to determine whether the steady state is locally stable. We leave details of the procedure into the online appendix.[10]

Step 2. We let the economy reach (v, π) at period 0 and let it be hit by an unanticipated shock so that before period-1 pairwise meetings it has a distribution of money different than π. Then we compute a *transitional equilibrium* $\{v_t, \pi_{t+1}\}_{t=1}^{\infty}$ starting from that period-1 distribution and approaching a post-shock steady state (v', π') (i.e., $(v_t, \pi_t) \to (v', \pi')$ as $t \to \infty$). If the post-shock average money holdings M' are equal to M then $(v', \pi') = (v, \pi)$; otherwise we seek (v', π') so that neutrality applies to (v', π') and (v, π). Because of indivisibility of money, *neutrality* means that $\Pi'(m) \equiv \sum_{x \leq m} \pi'(x) \approx \bar{\Pi}(mM/M')$ and $v'(m) \approx \bar{v}(mM/M')$, where $\bar{\Pi}(.)$ is the linear interpolation of the mapping $m \mapsto \Pi(m) \equiv \sum_{x \leq m} \pi(x)$ and $\bar{v}(.)$ is the linear interpolation of the mapping $m \mapsto v(m)$; we choose sufficiently large M/Δ and B/Δ (as detailed below) so that neutrality applies well, i.e., these approximations are sufficiently accurate.

Our algorithm to find a steady state is essentially an iteration on the mappings implied by (3)-(8). The algorithm is standard and, as all other algorithms, details and related codes are given in the online appendix.[11] Our algorithm to find a transitional equilibrium uses an approximation treatment; that is, (v', π') is reached after T

[10] The website is taozhu.people.ust.hk/nonneutrality.htm.

[11] For iterations in all algorithms, we stop when the two-round difference is less than 10^{-8}.

periods for a sufficiently large T. This algorithm makes sense only if (v', π') is locally stable so it is applied only after we confirm local stability of (v', π').

In the transitional equilibrium $\{v_t, \pi_{t+1}\}_{t=1}^{\infty}$, *the output response*, *the price response*, and *the markup response* at period t are measured by *the output increase* $Y_t/Y-1$, *the price increase* $P_t/P-1$, and *the markup increase* $\Upsilon_t/\Upsilon-1$, respectively. These and related statistics are defined as follows. Let $\left(y_t\left(m^b, m^s\right), \mu_t\left(.; m^b, m^s\right)\right)$ be the equilibrium meeting outcome at period t between a buyer with m^b and a seller with m^s. Then the expected payment and the expected price in the meeting are

$$d_t\left(m^b, m^s\right) = \sum_d \mu_t\left(d; m^b, m^s\right) d \text{ and } p_t\left(m^b, m^s\right) = d_t\left(m^b, m^s\right)/y_t\left(m^b, m^s\right), \quad (9)$$

respectively. Following Lagos, Rocheteau, and Wright [23], we interpret

$$\upsilon_t(m^b, m^s) = \frac{\sum_d \mu_t\left(d; m^b, m^s\right) \beta[v_{t+1}\left(m^s + d\right) - v_{t+1}\left(m^s\right)]}{c\left(y_t\left(m^b, m^s\right)/A\right)} \quad (10)$$

as the expected markup in the meeting.[12] We define aggregate output at t as

$$Y_t = 0.5 \sum_{m^b, m^s} \pi_t\left(m^b\right) \pi_t\left(m^s\right) y_t\left(m^b, m^s\right), \quad (11)$$

the average price at t as

$$P_t = \sum_{m^b, m^s} \pi_t\left(m^b\right) \pi_t\left(m^s\right) p_t\left(m^b, m^s\right), \quad (12)$$

and the average markup at t as

$$\Upsilon_t = \sum_{m^b, m^s} \pi_t\left(m^b\right) \pi_t\left(m^s\right) \upsilon_t(m^b, m^s). \quad (13)$$

With the time indices dropped, statistics in (9)-(13) represent their counterparts at the steady state (v, π).

Now we describe parameter values used in our analysis. For nominal objects, (Δ, M, B), M/Δ and B/Δ should be sufficiently large so that when M is changed to some nearby M', neutrality applies well to pre-change and post-change steady states. By various experiments, $M/\Delta = 30$ and $B/\Delta = 150$ serve the purpose well and $(\Delta, M, B) = (1, 30, 150)$ is maintained throughout.[13] For real objects, the term A is a free parameter and we simply set $A = 1$. We let the annual discount rate be 4% so that when agents meet F rounds in the decentralized market per year,

$$\beta = 1/\left(1 + 0.04/F\right).$$

In our benchmark, we set $\sigma = 1$ ($u(y) = \ln(y + \omega)$), $\eta = 1$ (the Frisch elasticity of labor supply is unity), and $F = 4$ (a period is a quarter). We consider two reference

[12] To understand the interpretation, rewrite the seller's surplus as $\frac{V}{Q} \cdot Q - C(Q)$, where $V = \sum_d \mu_t\left(d; m^b, m^s\right) \beta[v_{t+1}\left(m^s + d\right) - v_{t+1}\left(m^s\right)]$, $Q = c\left(y_t\left(m^b, m^s\right)/A\right)$, and $C(Q) = Q$. In this expression, the seller supplies Q, i.e, his present utility loss due to production, for exchange with V, i.e., his future utility gain due to the monetary payment, under the price V/Q. Treating the seller's surplus as his profit, $\upsilon_t(m^b, m^s)$ is the conventional price-marginal cost markup.

[13] We run some experiments with $M/\Delta = 300$ and $B/M = 5$ and observe rather limited effect on the key results.

values for the buyer's surplus share θ, 1 and the markup-determined share θ^{Υ}; the latter value, as proposed by Lagos, Rocheteau, and Wright [23], is chosen to match empirical evidences on markups. In our study, we target 1.39 as the average markup value Υ in (v, π) given other parameters.[14]

If one interprets the buyer's endowment of goods as a device that makes the buyer's reservation value in a pairwise meeting well defined when $\sigma \geq 1$, then ω should be sufficiently small. Alternatively and literally, the endowment may be something that can at least partially substitute for the seller's production and that may be related to the non-market activity or home production. We adopt the literal interpretation for two reasons: it is more realistic and while ω does not affect the basic output-response pattern, it does affect the price-response pattern. To ease exposition, we present our results by setting $\omega = 0.1$. This implies that home production is roughly equal to 10% of GDP; in the model, per capita real GDP is half of the average output and it turns out to be decreasing in ω. We discuss in section 4 about how ω affects the price-response pattern and it would be clear there that $\omega = 0.1$ is a conservative choice for the purpose to generate nominal rigidity.

3 Critical roles of a steady-state property and decentralized trade

Following the two-step procedure of numerical analysis given in section 2, we let the economy reach some steady state (v, π) at period 0. In our analysis, we use an imaginary period-1 shock to illustrate critical roles of a steady-state property and decentralized trade in determining the output response after a shock hits the economy; along this line, we illustrate how the output response may be affected by certain parameters. The imaginary shock does not change the stock of money but transforms the distribution π to $\pi_1^1(\pi)$ by dispersing (stretching) π or to $\pi_1^2(\pi)$ by squeezing π; the construction of $\pi_1^1(\pi)$ and $\pi_1^2(\pi)$ is borrowed from Wallace [38].[15]

[14]Citing the statistics in Faig and Jerez [13], Lagos, Rocheteau, and Wright [23] suggest that 1.39 be a suitable target of the average markup. In a recent work, De Loecker and Eeckhout [11] provide the average markup based on the estimated marginal costs for the US economy since 1950 and 1.39 is at the high end of the time series before 2000.

[15]Provided that the initial distribution of money is π, first assign each agent some additional money. An agent with m units of money is assigned $\min\{\lceil a(m)\rceil - 1, B - m\}$ with probability $p(m) = \lceil a(m)\rceil - a(m)$ and $\min\{\lceil a(m)\rceil, B - m\}$ with probability $1 - p(m)$, where $C \geq 0$ and C_0 are fixed numbers, $a(m) = \max\{0, C_0 + C \cdot m\}$, and $\lceil a(m)\rceil$ is the smallest integer not less than $a(m)$. Next, given that $M' - M$ is the total amount of assigned money, remove each unit of money independently from the economy with probability $1 - M/M'$. We associate $\pi_1^1(\pi)$ and $\pi_1^2(\pi)$ with some $C_0 < 0$ and $C_0 > 0$, respectively. In exercises below, we set $(C_0, C) = (-2, 0.1)$ for $\pi_1^1(\pi)$ and $(C_0, C) = (2, 0)$ for $\pi_1^2(\pi)$.

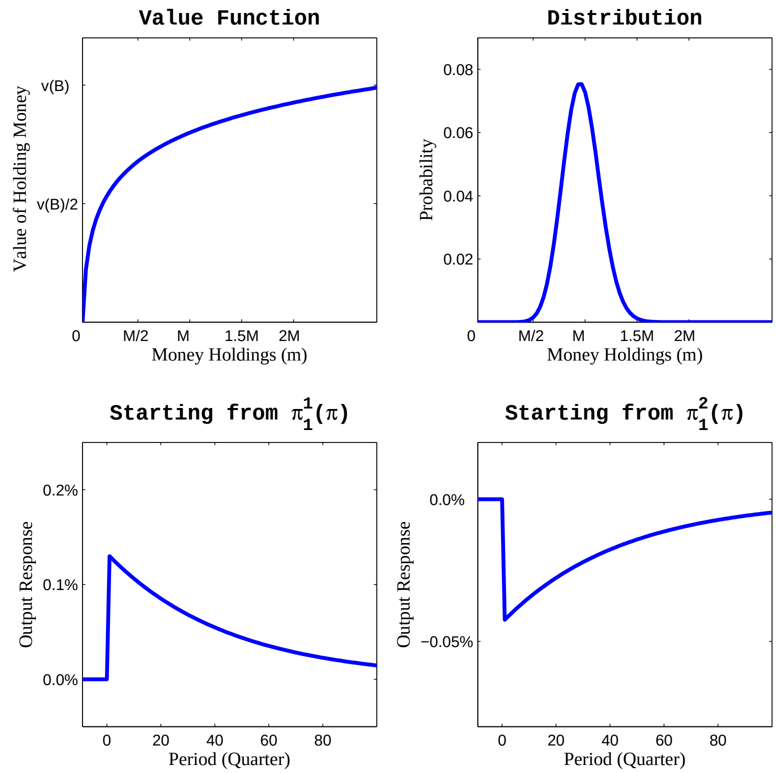

Figure 1: First row: steady-state value function and distribution; second row: output paths starting from $\pi_1^1(\pi)$ and $\pi_1^2(\pi)$.

Critical steady-state property

We begin with computed results for $(\sigma, \eta, F, \theta) = (1, 1, 4, 1)$. In Figure 1, the first row displays the steady-state value function v and the steady-state distribution π.[16] The second row displays the output responses along the transitional equilibria starting from $\pi_1^1(\pi)$ and from $\pi_1^2(\pi)$, respectively. Starting from $\pi_1^1(\pi)$, the output response is positive, significant, and persistent. Starting from $\pi_1^2(\pi)$, the output response is negative and much less significant.

We proceed by noting an important observation: substituting v for v_{t+1} we obtain a good approximation to the period-t output set $\mathbf{y}_t = \{y_t(m^b, m^s) : 0 \leq m^b, m^s \leq B\}$, the period-$t$ payment set $\mathbf{d}_t = \{d_t(m^b, m^s) : 0 \leq m^b, m^s \leq B\}$, and the period-$t$ price set $\mathbf{p}_t = \{p_t(m^b, m^s) : 0 \leq m^b, m^s \leq B\}$ because v_{t+1} is very close to v. In other words, people's incentives to trade in the transitional equilibrium are very close to their incentives to trade in the steady state. So the sets $\mathbf{y}_t$, $\mathbf{d}_t$, and $\mathbf{p}_t$ (calculated

[16]There is no result for uniqueness of a steady state. However, even though we choose many different initial values, our algorithm always converges to the same steady state.

when the future value of money is given by v_{t+1}) are all well approximated by their steady-state counterparts, denoted $\mathbf{y}$, $\mathbf{d}$, and $\mathbf{p}$ (calculated when the future value of money is given by v). In the present exercise, for example, the largest deviation of pairwise output $y_1\left(m^b, m^s\right)$ from $y\left(m^b, m^s\right)$ is around 0.004%; a deviation of x_1 from x is defined as $|x_1/x - 1|$.[17] Hence, the output responses in Figure 1 are essentially driven by the differences between post-shock distributions and π.

To see why a distribution different from π may drive a significant output response, we display the steady-state output set $\mathbf{y}$ in Figure 2. The top graph shows $\mathbf{y}$ in the three-dimension space. The two graphs at the bottom show two sorts of output curves that help to better visualize $\mathbf{y}$: the left graph consists of curves of the first sort each of which tells how $y\left(m^b, m^s\right)$ varies with m^s for a fixed m^b; the right graph consists of curves of the second sort each of which tells how $y\left(m^b, m^s\right)$ varies with m^b for a fixed m^s. Each curve of the first sort exhibits strong convexity, saying that when there is a marginal reduction in the seller's wealth, the increment in consumption received by a buyer is much larger if the seller is poorer. Because $\pi_1^1(\pi)$ is obtained from dispersing π, it follows that $Y\left(m^b, \pi_1^1(\pi)\right) > Y\left(m^b, \pi\right)$ all m^b, where $Y\left(m^b, h\right) = \sum h(m^s)\, y\left(m^b, m^s\right)$ is the average consumption for a buyer holding m^b under the distribution h provided that pairwise output is determined by $\mathbf{y}$. This need not imply $Y\left(\pi_1^1(\pi)\right) > Y(\pi)$, where $Y(h) = 0.5 \sum h\left(m^b\right) Y\left(m^b, h\right)$ is the aggregate consumption or output under the distribution h. Indeed, the buyer's average consumption is largely concave in his money holdings for a given h, i.e., $Y\left(m^b - 1, h\right) + Y\left(m^b + 1, h\right) < 2Y\left(m^b, h\right)$, as suggested by curves of the second sort.[18] But strong convexity is the dominant factor because of asymmetry in curvatures of two sorts of curves. That is, aggregate output would increase in the steady state if people's incentives to trade were not changed but π were dispersed to some h. This is the critical steady-state property that dictates the output-response patterns in display.

To examine the dependence of output response on parameters η, σ, and F, we keep $\theta = 1$ and conduct three exercises: (a) vary the elasticity of labor supply η from 1 to 4 (over some selected points) but maintain $(\sigma, F) = (1, 4)$; (b) vary the risk coefficient σ from 0.5 to 1.5 but maintain $(\eta, F) = (1, 4)$; and (c) vary the meeting frequency F from 4 to 365 but maintain $(\sigma, \eta) = (1, 1)$. Figure 3 displays output responses starting from $\pi_1^1(\pi)$ for some parameter values in these exercises; here and below we do not display responses from $\pi_1^2(\pi)$ because they remain insignificant.

In exercise (a), a smaller η gives rise to a weaker output response. This is an anticipated finding. After all, labor is the only input in our model and a smaller η means that it is more costly for a seller to increase his labor supply in a meeting. We find that a smaller η reduces asymmetry between curvatures of two sorts of output

[17] The largest deviations of the pairwise payment $d_1\left(m^b, m^s\right)$ from $d\left(m^b, m^s\right)$ and of the pairwise price $p_1\left(m^b, m^s\right)$ from $p\left(m^b, m^s\right)$ are 0.008% and 0.004%, respectively.

[18] Although shapes of two sorts of curves are quite intuitive, they are formed by many general-equilibrium forces. We cannot prove why one curve is convex and another is concave.

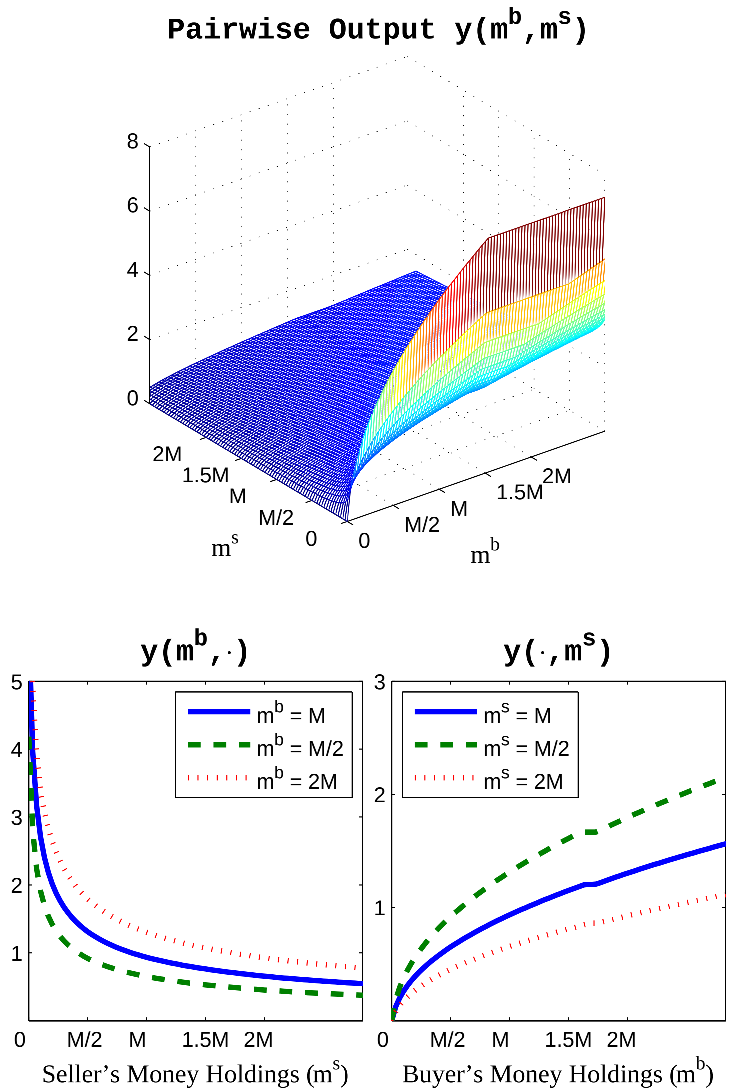

Figure 2: Steady-state pairwise output $\mathbf{y}_t = \left\{ y_t \left(m^b, m^s \right) : 0 \leq m^b, m^s \leq B \right\}$.

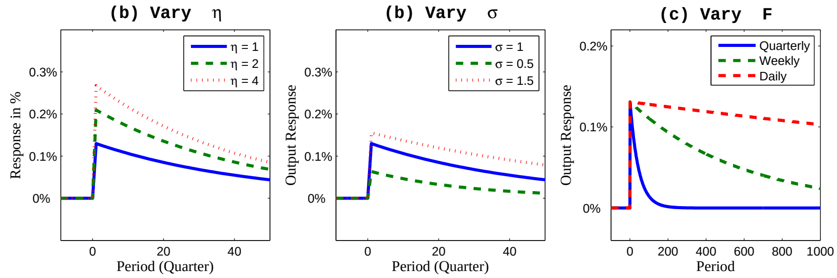

Figure 3: Transitional paths starting from $\pi_1^1(\pi)$ under (a) different η, (b) different σ and (c) different F.

curves mentioned above. Of course, one should keep in mind that the output curves might not capture all the structure of the set $\mathbf{y}$; in terms of the global structure, a smaller η would imply that there is a smaller degree of variation in $\mathbf{y}$.

In exercise (b), the output response is not much sensitive to σ and, in particular, when σ is in the range from 1 to 1.5. An increases in σ turns out to have two combined effects on the set $\mathbf{y}$: one is equivalent to increasing η and another is equivalent to a convex transformation of meeting outputs.[19] If σ moves up to 1, the two effects reinforce each other and, hence, increases the variation in $\mathbf{y}$. If σ moves up away from 1, then the two effects offset each other. Overall, the role of σ is rather minor, at least when it is close to or above unity, compared with the role of η in affecting the output response.

In exercise (c), F does not affect the peak output response (occurring at period 1). With a larger F, the output response stays above a fixed level for more periods. But if we measure the output response against a fixed time span (e.g., a quarter or a week), then the output response curve does not vary much with F. Remarkably, the annual nominal GDP approaches an upper bound when F moves up. This is because the value of a unit of money is increasing in F and so the amount of money spent in each round of pairwise meetings is decreasing.

In figures above and below, we only present transitional paths in the first 20 to 50 periods. Along a transitional path, aggregate output gradually declines to the post-shock steady-state level. The whole process is lengthy. When $F = 4$, depending on other parameter values, it takes about 35 to 45 periods for the response to decline 50% from the peak and, from there it takes about another 35 to 45 periods to decline 50%; it takes 350 to 650 periods for the deviation of Y_t from Y to fall below 0.0001%.

[19]The utility function of y in (2) and the disutility function of l in (1) are equivalent to the utility function (of z) $(z+\omega)^{1-\sigma'}/(1-\sigma')$ and the disutility function (of l) $K_1 l^{1+1/\eta'}$ through the relationship $(1-\sigma)(1+1/\eta') = (1-\sigma')(1+1/\eta)$, $z+\omega = K_2(y+\omega)^{\frac{1-\sigma}{1-\sigma'}}$, and $z = Al$ for some constant K_1 and K_2. Fix η and $\sigma' < \delta$ if $\delta < 1$ and $\sigma' > \delta$ if $\delta > 1$. When $\sigma < 1$ increases or $\sigma > 1$ decreases toward unity, one observes two effects on output y (associated with the former pair of functions): (i) η' decreases so does output z (associated with the latter pair of functions), and (ii) $\frac{1-\sigma'}{1-\sigma}$ increases so $(z+\omega)^{\frac{1-\sigma'}{1-\sigma}}$ is more convex.

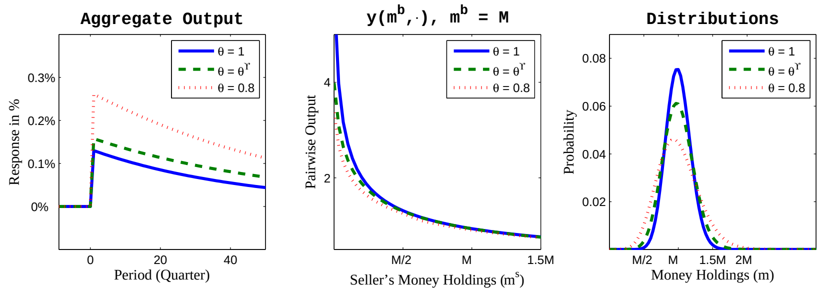

Figure 4: Left: transitional paths starting from $\pi_1^1(\pi)$ under different θ; mid and right: steady-state output curves and distributions under different θ. $(\sigma, \eta, F) = (1, 1, 4)$.

Markup-determined buyer's surplus share

Given a tuple (σ, η, F) in exercises (a), (b), and (c) above, there is a range of buyer's surplus shares, including the markup-determined share θ^{Υ}, which preserve the aforementioned critical steady-state property and, hence, the output response patterns.

Here we first discuss some details of the output response when θ is varied. For illustration, we set $(\sigma, \eta, F) = (1, 1, 4)$. The left graph of Figure 4 displays output responses starting from $\pi_1^1(\pi)$ for $\theta \in \{1, \theta^{\Upsilon}, 0.8\}$, $\theta^{\Upsilon} = 0.912$. Two forces are in play. As displayed in the mid graph of Figure 4, a smaller θ leads to a less curvature in an output curve of the first sort (this tends to dampen the output response); as displayed in the right graph of Figure 4, a smaller θ leads to a more dispersed steady-state distribution (this tends to amplify the output response).

The interaction of these two forces makes the output response sensitive to how the underlying shock disperses the distribution of money. Indeed, we can construct a $\pi_1^1(\pi)$ differently so that the order in the left graph of Figure 4 is reversed: the peak output response is 0.50% for $\theta = 1$ and 0.28% for $\theta = 0.8$.[20] To understand the reversal, we note that $var(\pi_1^1(\pi)) - var(\pi)$ increases from 4.7 to 10.1 when θ falls from 1 to 0.8 for the original $\pi_1^1(\pi)$, while $var(\pi_1^1(\pi)) - var(\pi)$ ranges from 6 to 7 under various θ for the differently-constructed $\pi_1^1(\pi)$; that is, when π itself is more dispersed (accompanying a smaller θ), the imaginary shock underlying the original $\pi_1^1(\pi)$ induces a much stronger dispersing effect than the shock underlying the differently-constructed $\pi_1^1(\pi)$.

Next we report some details pertaining to θ^{Υ} and markups. First, θ^{Υ} is decreasing in η, increasing in σ, and nearly constant in F for a fixed average markup Υ. Given $\Upsilon = 1.39$, θ^{Υ} moves down to 0.835 if η alone moves up to 4, and θ^{Υ} varies from 0.825 to 0.999 if σ alone varies from 0.5 to 1.5. Second, the average markup Υ is decreasing in η, increasing in σ, and nearly constant in F for a fixed θ. Given $\theta = 0.912$, Υ moves

[20] For this $\pi_1^1(\pi)$, let each agent with m units of money be hit by a shock: with probability $1 - \vartheta$, the agent keeps m; with probability ϑ, he draws $\varsigma(m)$ from a discrete uniform distribution on $\{-q, \ldots, 0, \ldots q\}$, where $q = \min(m, B - m)$, and his money holdings become $m + \varsigma(m)$. We use $\vartheta = 0.02$ in construction.

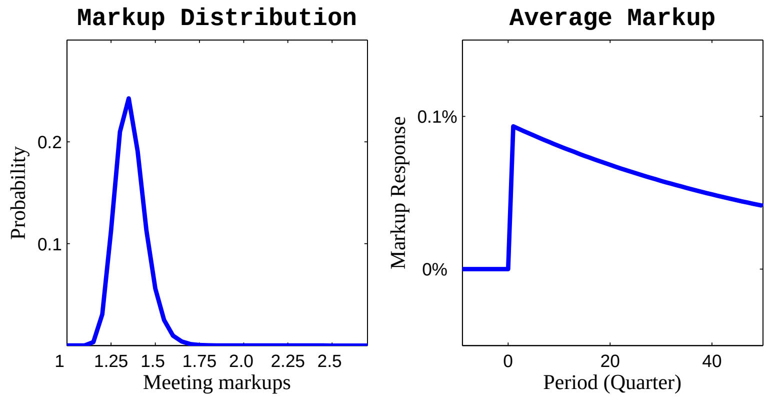

Figure 5: Left: steady-state distribution of meeting markups; right: response of average markup starting from $\pi_1^1(\pi)$. $\theta = \theta^\Upsilon$.

down to 1.20 if η alone moves up to 4, and Υ varies from 1.19 to 1.87 if σ varies from 0.5 to 1.5. Third, the heterogeneity of meeting markups in the steady state, as displayed the left graph of Figure 5, is consistent with what has been estimated by Hall [18]. Lastly, the positive comovement of the average markup Υ_t and aggregate output Y_t for $\theta = \theta^\Upsilon$, as displayed in the right graph of Figure 5, is consistent with what has been found by Nekarda and Ramey [31].

Roles of decentralized trade

Next we turn to the role of decentralized trade. Decentralized trade certainly matters for persistence of the output response because it slows down the dispersion (diffusion) of money redistributed by the shock. For its contribution to significance of the output response, we modify the basic model by replacing pairwise meetings with a centralized meeting. In this modified version, each agent has the equal chance to be a buyer or a seller in a competitive market. Each agent takes the price of money ϕ_t as given. He trades with the market a lottery μ for monetary payments and a quantity y for goods such that the expected monetary payment implied by the lottery μ is y/ϕ_t.

Let v_t and π_t be the same as in the basic model. Given π_0, an equilibrium is a sequence $\{v_t, \phi_t, \pi_{t+1}\}_{t=0}^{\infty}$ satisfying standard conditions on the law of motion, the recursive relation between value functions, and the market clearing; details of these conditions are given in the appendix. A triple (v, ϕ, π) is a steady state if $\{v_t, \phi_t, \pi_{t+1}\}_{t=0}^{\infty}$ with $(v_t, \phi_t, \pi_{t+1}) = (v, \phi, \pi)$ all t is an equilibrium.

We compute a steady state (v, ϕ, π) and a transitional equilibrium from $\pi_1^1(\pi)$ for $(\sigma, \eta, F) = (1, 1, 4)$. The output response, displayed in the left graph in Figure 6, is transient, negative and insignificant. To understand this pattern, we refer to the middle and right graphs in Figure 6 obtained from (v, ϕ, π): the middle graph has a *buyer's consumption curve* that tells how the buyer's consumption varies with his money holdings and the right graph has a *seller's production curve* that tells how the

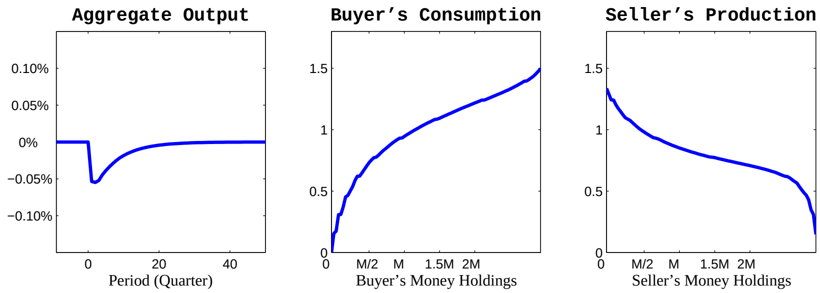

Figure 6: Left: centralized market's output response from $\pi_1^1(\pi)$; middle and right: buyer's consumption curve and seller's production curve.

seller's production varies with his money holdings.

According to curvatures of these two curves, if ϕ_1 is the same as ϕ, then dispersing π to $\pi_1^1(\pi)$ tends to raise the period-1 aggregate production above the steady-state level and reduce the period-1 aggregate consumption below the steady-state level; the influence from agents with holdings greater than $2M$ may be ignored because the proportion of these agents is very small. To clear the market, ϕ_1 must rise above ϕ (the goods price must fall) so that the period-1 buyer's consumption curve is shifted up and the period-1 seller's production curve is shifted down. Examining either curve, the dispersing effect and the shifting effect offset each other, leading to an insignificant output effect. Starting from $\pi_1^2(\pi)$, the output response is insignificant by the same reason (while it turns to be positive).

4 One-shot regressive money injection

In this section, we replace the imaginary shock in section 3 with a monetary shock. Specifically, the government injects money from period 1 to N, raising the stock of money from M to M'. As indicated in section 2, we look for the post-shock steady state (v', π') such that neutrality applies to (v, π) and (v', π'). Given this neutrality, aggregate output Y' and the average price P' at the steady state (v', π') are very close to Y and PM'/M, respectively.[21] As in section 3, the period-t output, payment, and price sets $\mathbf{y}_t$, $\mathbf{d}_t$, and $\mathbf{p}_t$ are all well approximated by their counterparts $\mathbf{y}'$, $\mathbf{d}'$, and $\mathbf{p}'$ at the post-shock steady state. So if the injection induces a distribution before period-1 pairwise meetings more dispersed than π', then Y_1 increases relative to Y' and, hence, relative to Y. Recall that $\Pi'(m) \approx \bar{\Pi}(mM/M')$, implying that we should consider a regressive money injection.

To discipline us in selecting the degree of regressiveness of an injection, we let it be determined by an endogenized limited participation as follows. At period $t \in \{1, \ldots, N\}$, agents are entitled to buy lotteries with money before pairwise meetings:

[21] For exercises in this section, $M' = 1.01M$, the deviation of Y' from Y is no greater 0.007%, and the deviation of P' from PM'/M is no greater than 0.09%.

if an agent pays x units of money, then he receives $2x$ units of money with probability $\chi_t \in (0.5, 1]$ that is set by the government; otherwise he receives no money. Given χ_1, $(\chi_{t+1} - 0.5) / (\chi_t - 0.5) \equiv \rho_t \in (0, 1)$ for $t \in \{1, \ldots, N-1\}$ if $N > 1$. Strictly concave value functions and nondegenerate distributions imply a limited participation in that richer agents tend to spend more on lotteries and, hence, receive a larger portion of injected money.[22]

The equilibrium conditions at period $t \geq N+1$ are the same as those in section 2. At period $t \in \{1, \ldots, N\}$, the equilibrium conditions involve (v_t, π_t) as usual and the distribution $\breve{\pi}_t$ of money after the injection but before pairwise meetings; details of these conditions are given in the appendix.

We set $N \in \{1, 5\}$ in our numerical analysis: $N = 1$ is the benchmark while $N = 5$ may mimic the familiar AR(1) process of monetary shocks. When $N = 1$ we seek a suitable lottery-winning probability χ_1 to meet $M' = 1.01M$. When $N = 5$, we set $\rho_t = \rho = 0.65$ and again we seek a χ_1 to meet $M' = 1.01M$; $\rho = 0.65$ turns out to imply that the amount of money injected at $t+1 < N$ is around half of the amount of money injected at $t \geq 1$. Figure 7 displays the output and price responses along the transitional equilibrium for $(\sigma, \eta, F) = (1, 1, 4)$ and $\theta \in \{1, \theta^{\Upsilon}\}$, $\theta^{\Upsilon} = 0.912$.

We begin with the output responses. The output responses when $N = 1$ conform to the pattern of the output response along the transitional equilibrium starting from $\pi_1^1(\pi)$ in section 2. When $N = 5$, output expands with a declining rate as the total money stock increases; the reason is simple—the injection at $t+1$ reinforces the dispersion on the distribution of money made by the injection at t but the reinforcing effect declines as the injection rate declines.

The output response for $\theta = \theta^{\Upsilon}$ is less significant but more persistent than for $\theta = 1$. For this, we refer to the two accompanying forces for θ decreases indicated in section 3. The dispersion from π to $\breve{\pi}_1$ induced by an injection here is only slightly more for $\theta = \theta^{\Upsilon}$ than for $\theta = 1$. When $N = 1$, $var(\breve{\pi}_1) - var(\pi)$ is 9.8 for $\theta = 1$ and 9.7 for $\theta = \theta^{\Upsilon}$.

Other lessons in section 3 about the output responses also remain valid. When we only change η, a larger η leads to a larger output response; if we raise η to 2, for example, the output increase is about 1.7 times the value for $\eta = 1$. When we only change σ, we observe small changes in the output response except that there is more persistence as σ increases. Moreover, when we only change F, the peak output response does not vary and, independent of F, the 1% accumulative increase in the money stock can induce an accumulative increase in output over 20 quarters that

[22] Recall that there is an upper bound B on the individual holdings. This bound affects the lottery-purchasing decision for agents whose holdings are close to the bound (an agent with holdings B does not buy any lottery). But these effects have little influence on the output response because the measure of agents with such holdings are very small. In the basic model with benchmark parameter values, for example, $\pi(B) = 4.28 \times 10^{-42}$ and $\pi(B-1) = 8.00 \times 10^{-42}$ in the steady state. In general, we need a finite B for numerical exercises. In this model, if B has any influence on our result, the influence is to dampen the output response; for, the upper bound being relaxed, the money injection would only disperse the distribution of money further.

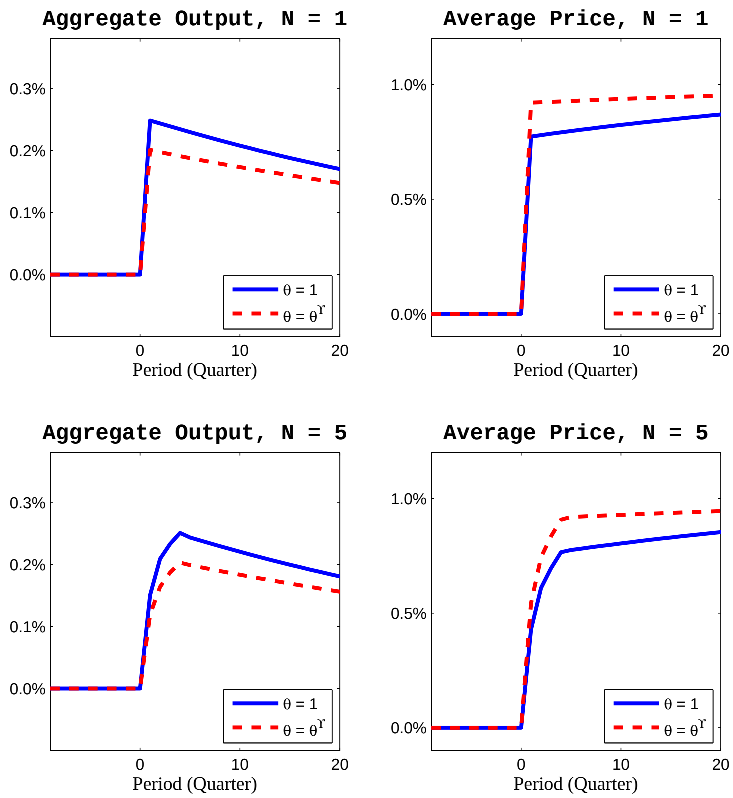

Figure 7: Output and price responses after a 1% money injection. Upper row: $N = 1$; bottom row: $N = 5$.

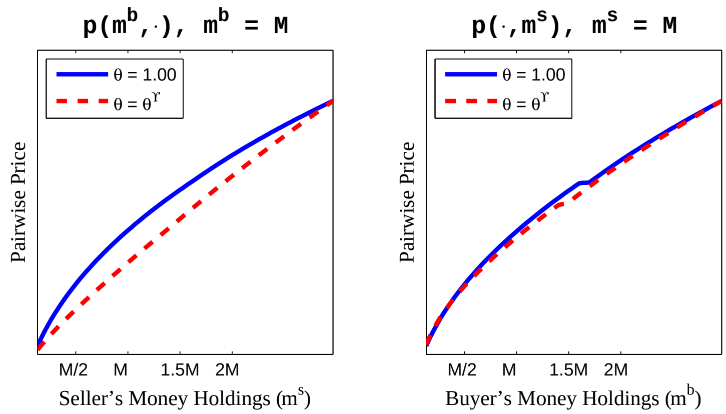

Figure 8: Transition paths and steady-state price curves, under different θ.

exceeds more than 4% of quarterly output in the pre-shock steady state.

Next we turn to the price responses. To make sense of the response patterns in Figure 7, we display in Figure 8 two sorts of price curves obtained from the steady-state price set $\mathbf{p}'$ (recall that $\mathbf{p}_t$ is well approximated by $\mathbf{p}'$). In the left graph, a price curve from a post-shock steady state tells how the pairwise price $p'\left(m^b, m^s\right)$ varies with m^s for the fixed $m^b = M$; in the right graph, a price curve from a post-shock steady state tells how $p'\left(m^b, m^s\right)$ varies with m^b for the fixed $m^s = M$. For comparison, price curves in each graph are rescaled so that the highest prices appear to be the same. Because the price curves are concave, P_t falls below P' because the injection is regressive (recall $\Pi'(m) \approx \bar{\Pi}(mM/M')$ and $P' \approx PM'/M$). This explains the nominal rigidity shown in Figure 7. Moreover, the price response is more sluggish for $\theta = 1$ than for $\theta = \theta^{\Upsilon}$ in Figure 7 because the former price curves are more concave.

The shapes of price curves are largely determined by θ, σ, and ω. Specifically, a larger θ, a smaller σ, or a larger ω tends to bend a price curve down.[23] For $\theta = 1$, if we vary σ or ω or both, we may change how concave a price curve is (but do not change concavity itself) so the degree of nominal rigidity may vary; in the Figure-7 exercise, for example, the period-1 price response is 0.61% at $\omega = 0.5$. For θ away from unity, we observe a clear trend about the price curve's shape by fixing σ close

[23]That is, with a larger θ, a smaller σ, or a larger ω, $p'\left(m^b, m^s+1\right) - p'\left(m^b, m^s\right)$, the extra price received by a seller with m^s+1 relative to a seller with m^s in meeting a buyer with m^b tends to increase less from $p'\left(m^b, m^s\right) - p'\left(m^b, m^s-1\right)$; and $p'\left(m^b+1, m^s\right) - p'\left(m^b, m^s\right)$, the extra price paid by a buyer with m^b+1 relative to a buyer with m^b in meeting a seller with m^s tends to increase less from $p'\left(m^b, m^s\right) - p'\left(m^b-1, m^s\right)$. The role played by θ is intuitive: if θ is larger, then the seller with m^s+1 receives a smaller extra benefit and the buyer with m^b+1 receives a larger extra benefit. Regarding σ and ω, they may work through the curvature of the value function v'. A smaller σ or a larger ω tends to make v' flatter (by way of making the function $y \mapsto u(y)$ flatter). Intuitively, if v' is flatter, then it may rely less on the extra price to deliver the extra benefit to the seller with m^s+1 and it may be more costly for the buyer with m^b+1 to pay an extra price.

to or above unity and varying ω. If ω moves up from a certain value, the price curve becomes more concave, implying that a more sluggish price response; in the Figure-7 exercise, the period-1 price response is 0.73% for $\theta = \theta^{\Upsilon}$ at $\omega = 0.5$.[24] If ω moves down, then the price curve easily becomes convex; in the Figure-7 exercise, one sees a price overshooting for $\theta = \theta^{\Upsilon}$.

To sum up, the output-response pattern following an injection is determined by the shape of the steady-state output set $\mathbf{y}'$ and the price-response pattern is determined by the shape of the steady-state price set $\mathbf{p}'$. Over the range of parameter values that are examined, the shape of the output set is stable; such stability holds when ω is part of the varied parameters while we omit details above to simplify exposition.[25] Over the same range of parameter values, the shape of the price set has an apparent dependency on ω for θ is away from unity. As GDP is decreasing in ω, nominal rigidity co-occurs with the real expansion when ω reaches a level that home production is not too small compared with GDP.[26]

It may be worth reiterating the basic logic behind the co-occurrence of the real expansion and nominal rigidity: $\mathbf{y}_t$ and $\mathbf{p}_t$ in the transition path are well approximated by $\mathbf{y}'$ and $\mathbf{p}'$, the injection increases the mass of meetings with $y'\left(m^b, m^s\right) > Y'$ and $p'\left(m^b, m^s\right) < P'$ and the mass of meetings with $y'\left(m^b, m^s\right) < Y'$ and $p'\left(m^b, m^s\right) > P'$, but shapes of $\mathbf{y}'$ and $\mathbf{p}'$ dictate that effects of the former meetings dominate. Clearly, the co-occurrence does not mean causality in either way. In this regard, our study retells the lesson taught by Lucas [26] in his celebrated study of the Phillips curve; that is, it is misleading to draw a causal relationship based on an observed output-price correlation.

We complete this section with two remarks. First, there is a non-vertical Phillips curve in our model. If the injection increases the money stock by a% and a is not very large (say, $a \leq 5$), then the output and price responses are about a times values for the corresponding 1%-increase case. Second, the average markup Υ_t moves up with output Y_t for $\theta = \theta^{\Upsilon}$ by the same pattern as in the right graph of Figure 5. So an expansionary monetary shock drives up both output and markups in the presence of nominal rigidity. It is interesting to note that with imposed nominal rigidity, the New Keynesian models typically predict countercyclical markups following an expansionary monetary shock.

[24] Of course, θ^{Υ} varies with ω. We find that θ^{Υ} is decreasing in ω. When ω alone varies from 10^{-4} to 0.5, θ^{Υ} varies from 0.980 to 0.826.

[25] The value of ω may have some effect on the peak output response. In the Figure-7 exercise, when ω alone varies from 10^{-4} to 0.5, the peak output response varies from 0.27% to 0.14% for $\theta = 1$ and varies from 0.37% to 0.23% for $\theta = \theta^{\Upsilon}$.

[26] We note in section 2 that the ratio of home production to GDP is about 10% at $\omega = 0.1$. For the U.S from 1965 to 2010, home production on average is equivalent to more than 30% of GDP (see Bridgman et al. [5]). According to an official statistics from U.K. (see Webber et al.[39]), home production is equivalent to more than 50% of GDP from 2005 to 2014. We choose $\omega = 0.1$ to counter concerns that home production may not give a perfect substitute to the goods exchanged from the market and that sellers may also engage in home production.

5 Bonds and nominal interest rate

Does a money injection have a liquidity effect? More generally, how does the nominal interest rate comove with the money stock, output, and inflation rate? With money as the unique asset, the basic model leaves no room to address these issues that are pivotal in much of the literature on monetary shocks. Following a standard approach used in Lucas [27] to study liquidity effects, we modify the basic model by letting the government issue one-period discount bonds that are financed by inflation. The simple setting allows us to examine the dynamics of the interest rate and inflation rate following a money injection; moreover, it helps to address the concern that the basic model may exaggerate the output effect of a money injection because in the absence of a non-monetary store of value, some buyers may overspend their gains in wealth from the injection.

The modified model

Each period t consists of two stages, 1 and 2. At stage 1, the government issues D_t amount of bonds on a competitive market; each unit of bonds automatically turns into one unit of money at the start of period $t+1$. We adopt the following setting in Zhu and Wallace [46] to represent discount bonds for indivisible nominal assets. That is, each agent entering the bond market with m units of money can choose a probability measure $\hat{\mu}$ (a lottery) defined on the set $\Xi = \{\zeta = (m', w') \in B_\Delta \times B_\Delta : w' \geq m'\}$ that satisfies

$$\sum_{\zeta'=(m',w')} \hat{\mu}(\zeta') \cdot \left(m' + p_t^B (w' - m')\right) \leq m, \tag{14}$$

where p_t^B, interpreted as the price of bonds, is taken as given by the agent and $\hat{\mu}(\zeta')$ is the probability for the agent to leave the bond market with m' units of money and $w' - m'$ units of bonds. In equilibrium, p_t^B clears the bond market. At stage 2, agents are matched in pairs as in the basic model. In pairwise meetings, each agent can observe his meeting partner's portfolio, but bonds are illiquid and money is the unique payment method.

Because of interest payments, the total money stock would increase over time. If money were divisible, then we would simply normalize the state of an agent at period t right before issuance of new bonds as mM_t/M_t^+, where m is the agent's money holdings at that time, M_t^+ is the money stock at that time, and M_t is the difference between M_t^+ and period-t interest payments $D_{t-1}\left(1 - p_{t-1}^B\right)$. Because nominal assets are indivisible, mM_t/M_t^+ need not be an integer. So we follow Deviatov and Wallace [12] by assuming that right before issuance of new bonds, each unit of money automatically disintegrates with the probability $\delta_t = 1 - M_t/M_t^+$, turning the money stock back to M_t.[27] Setting $M_0 = M$, we have $M_t = M$, all t.

[27]If we apply this disintegration to divisible money, an agent with m units of money before disintegration holds mM_t/M_t^+ after disintegration, exactly equivalent to the normalization without

Let v_t and π_t be the value function and the distribution of money after disintegration but before bonds issuance at period t (so $\sum_m \pi_t(m)\, m = M$) and let $\hat{\pi}_t$ be the distribution of portfolios right before pairwise meetings. Given π_0 and $\{D_t\}_{t=0}^{\infty}$, an equilibrium is a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}, p_t^B\}_{t=0}^{\infty}$ satisfying standard conditions on the laws of motion, the recursive relations between value functions, and the bond market clearing; details of these conditions are given in the appendix. A tuple $\left(v, \hat{\pi}, \pi, p^B\right)$ is a steady state if $\{v_t, \hat{\pi}_t, \pi_{t+1}, p_t^B\}_{t=0}^{\infty}$ with $\left(v_t, \hat{\pi}_t, \pi_t, p_t^B\right) = \left(v, \hat{\pi}, \pi, p^B\right)$ all t is an equilibrium. In a steady state, it is necessary to have $D_t = D$ for some D.

We interpret the product of the period-t gross growth rate in the price level and M_t^+/M (the period-t gross growth rate in the money stock if there is no disintegration) as *the gross inflation rate* at $t \geq 1$, denoted $1 + \varphi_t$, i.e.,

$$1 + \varphi_t = \frac{1}{1 - \delta_t} \frac{P_t}{P_{t-1}}. \tag{15}$$

We interpret $i_t \equiv 1/p_t^B - 1$ as the nominal interest rate and, hence, $i_t - \varphi_t$ as the real interest rate at t. The real interest rate at steady state is nearly zero because interests are financed by inflation.

We apply the same monetary shock as in section 4. The economy reaches a steady state at period 0. At period $t \in \{1, ...N\}$, money is injected before issuance of new bonds;[28] $M' - M$ is the total amount of money injected over the N periods. Now M_t^+ is the period-t money stock after injection but before issuance of new bonds and, as above, $M_t = M_t^+ - D_{t-1}\left(1 - p_{t-1}^B\right)$ and $\delta_t = 1 - M_t/M_t^+$. Starting from period 1 the supply of bonds follows the process

$$D_t = D' - \psi_t \cdot \left(D' - D_{t-1}\right), \tag{16}$$

where $D_1 = D$, D' is the supply of bonds at the post-shock steady state, and the sequence $\{\psi_t\}$ governs how quickly the bonds-money ratio returns to the steady-state level.

In numerical analysis, we follow the two-step procedure described in section 2 and we also adopt parameter values in the basic model, including benchmark values for (σ, η, F). The value of D is chose so that the nominal interest rate at the pre-injection steady state is around 1.5%; given $M = 30$, $D = 29.45$ serves the purpose.

Liquidity effect and price puzzle

In the modified model, a money injection remains effective in inducing strong output responses; the injection generates a liquidity effect; and by way of the liquidity effect,

disintegration.

[28]If we rely on an open market operation to inject money (after the bonds issuance but before pairwise meetings), then we must give agents incentives to exchange bonds which they just purchased with money. One possibility is that agents know their types in pairwise meetings. As it turns out, such a money injection does not redistribute wealth sufficiently to generate a significant and persist output response.

Another sort of policy shock is to have a sudden change in the bond issuance. The shock does not redistribute wealth sufficiently either.

a simple channel generates a fall in the inflation rate. Some VAR studies find a fall in the inflation rate following an expansionary policy shock and refer to it as a price puzzle (see, e.g. Christiano, Eichenbaum, and Evans [9]). To present results, we set $(\sigma, \eta, F) = (1, 1, 4)$, $\theta \in \{1, \theta^\Upsilon\}$, where $\theta^\Upsilon = 0.913$.

To highlight the key points, we start with $N = 1$ and set $M' = 1.01M$ (as in the basic model, we seek a suitable χ_1 to meet this target). We choose $D' = 29.76 \approx DM'/M$ so that neutrality applies well to pre-shock and post-shock steady states.[29] We set $\psi_t = 0.5$ all t. Figure 9 presents transition paths. To facilitate the discussion, let Z_{t+1} be the average nominal wealth at the start of period $t+1$ in the transitional equilibrium; let Z be the average nominal wealth at the start of a period after the post-shock steady state is reached. As in the basic model, one gets a good approximation to period-t pairwise meeting outcomes in the transitional equilibrium by substituting the value of holding zZ/Z_{t+1} at the start of a period at the post-shock steady state for the value of holding z at the start of period $t+1$.[30]

Figure 9 shows that the injection has a liquidity effect. The nominal interest is around 0.5% at period 1 while it is around 1.5% in the pre-injection steady state. This may be explained as follows. If an agent carries j units of money into pairwise meetings in the post-shock steady state, then he receives a close service from carrying $j_t = jZ_{t+1}/Z$ units of money into period-t pairwise meetings in the transitional equilibrium. So $J_t = \kappa J Z_{t+1}/Z$ for some κ not far from one, where J_t and J are the average money holdings carried into period-t and steady-state pairwise meetings, respectively. But J is slightly greater than 1 and because Z_{t+1}/Z is around 1, $J_t - J = (\kappa Z_{t+1}/Z - 1)J$ is too small to absorb most of injected money.

Figure 9 shows that the injection drives down inflation. The inflation rate φ is around 1.5% in the pre-injection steady state. For $\theta = 1$, the inflation rate falls down to 1.3% immediately after the shock and later to 1.0%. For $\theta = \theta^\Upsilon$, the inflation rate increases slightly before falling down to 1.0% after two periods.

Why does the injection drive down inflation? Consider $\theta = 1$. One unit of money in pairwise meetings at period 1 is worth, in the approximation sense, Z/Z_2 units in pairwise meetings at the post-shock steady state. So $P_1 = \kappa_1 P' Z_2/Z = \kappa_1 P(M'/M)(Z_2/Z)$ for some κ_1. The term κ_1 reflects the effect on P_1 caused by the difference between π' and the distribution of nominal wealth following the injection; in the corresponding situation in the basic model, it is less than 1. By definition $Z_2 = M' + Di_1$ and $Z = M' + D'i'$, where $i' \approx i$ is the nominal interest rate in the

[29]Because of indivisibility of nominal assets, this D' brings output and the nominal interest rate in (v', π') closer to those in (v, π) than $D' = DM'/M$. For $\theta = 1$, deviations of Y' from Y, P'/M' from P/M, and $p^{B\prime}$ from p^B are 0.001%, 0.004%, and 0.02%, respectively. If we instead choose $D' = DM'/M$, the corresponding numbers are 0.1%, 1.3%, and 0.03%.

[30]In the basic model, if the stock of money M_{t+1} at period $t+1$ in the transitional equilibrium is not equal to the stock of money M' in the post-injection steady state (e.g., $t = 2$ when $N = 5$), the value of holding m at $t+1$ is approximated by the value of holding mM'/M_{t+1} at the post-injection steady state. If mM'/M_{t+1} or zZ/Z_{t+1} is not an integer, the steady-state value is taken from the linear interpolation of the relevant steady-state value function.

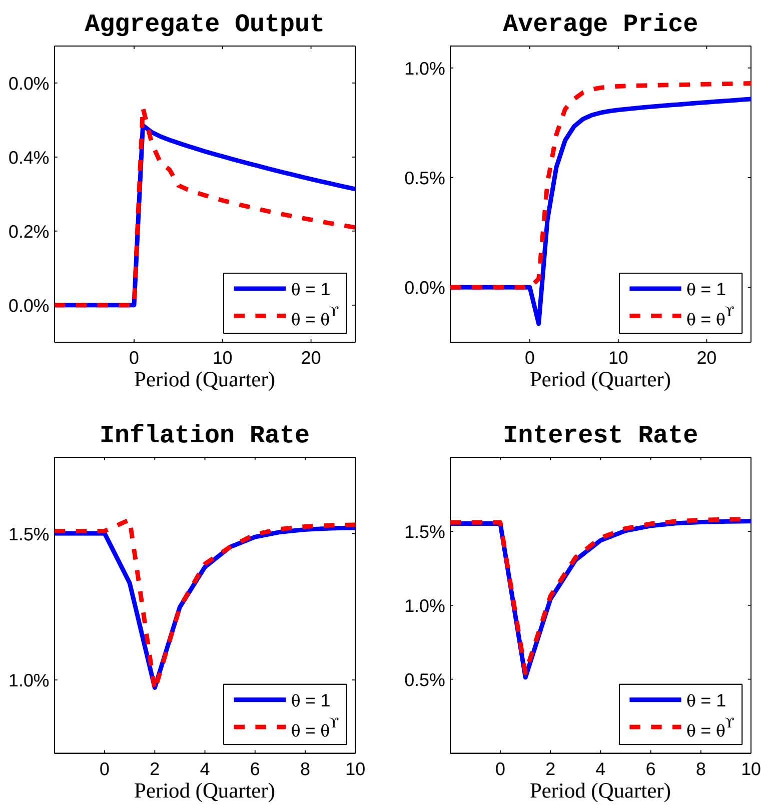

Figure 9: Transitional paths following a 1% money injection with $N = 1$, under different θ.

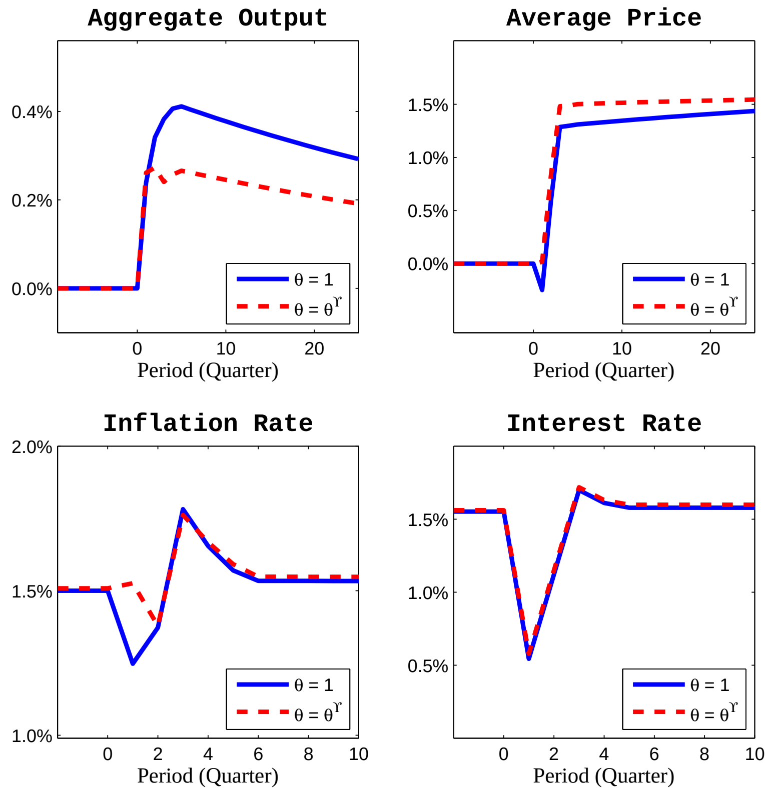

Figure 10: Transitional paths following a 1.667% money injection with $N = 1$, under different θ.

post-shock steady state. The difference in interest payments $D'i' - Di_1$ is around 0.3, implying $Z/Z_2 \approx 1.013$; that is, the effect on P_1 from the increase in the nominal stock is dominated by the effect on P_1 from the increase in the real value of money. Because δ_1 is equal to the disintegration rate δ in the pre-injection steady state, it follows that φ_1 is below φ. Similarly, $P_2 = \kappa_2 P(M'/M)(Z_3/Z)$ for some κ_2 (playing the same role as κ_1). Because the difference in interest payments $D'i' - Di_2$ is nearly zero, P_2 is close to $\kappa_2 P(M'/M)$ and, hence, P_2/P_1 is close to 1.01. But φ_2 is still below φ because δ_2 is much smaller than δ, a consequence of the substantial difference in interest payments $D'i' - Di_1$. The only difference between $\theta = 1$ and $\theta = \theta^\Upsilon$ is that κ_1 is larger and, hence P_1 is less stickier for $\theta = \theta^\Upsilon$ than for $\theta = 1$.

To emphasize, the reduction in interest payments $D'i' - Di_1$ at period 2 has a direct effect on δ_2 and an indirect effect on P_1 through δ_2 so that it affects inflation rates at periods 1 and 2. The general point is that if the steady-state inflation is driven by interest payments to government bonds, then the liquidity effect of an injection may easily drive down the inflation.

The modified model can generate richer and more interesting dynamics in real and nominal variables with a larger N. Staying close to the basic model, we let $N = 5$ and $\rho_t = \rho = 0.65$. We seek a suitable χ_1 so that $M'/M - 1 = 1.667\%$ and money injected at period 1 is around $0.01M$ (the nominal interest at period 1 then is down by the same magnitude as above). We set $D' = 29.966 \approx DM'/M$ and choose $\{\psi_t\}_{t=1}^{T}$ so that $D_t/M_{t-1} = D_{t-1}/M_{t-2}$ for $t < 3$ and $D_t = D'$ for $t \geq 3$. In this process, D_t/M_t exceeds the steady-state level at $t = 3$. Transitional paths are in Figure 10.

In Figure 10, the output response peaks at the last period of the injection (which is the same as in the basic model). The nominal interest rate falls by 1% initially and next rises up; it exceeds the steady-state level at $t = 3$ and then slowly falls back, which makes sense because D_t/M_t exceeds the steady-state level at $t = 3$ and then falls back to the steady-state level.

The movements of the inflation rate at periods 1 and 2 are the same as in the exercise with $N = 1$. The critical difference occurs at period 3: the inflation rate in the present exercise exceeds the steady-state level and then falls back. Notice that $D_t/M_t > D/M$ for $t \geq 3$ leads to $\delta_{t+1} > \delta$ (by affecting interest payments at period $t+1$), which, in turn, leads to $\varphi_{t+1} > \varphi$; $D_3/M_3 > D/M$ also leads to $\varphi_3 > \varphi$ (by affecting P_3 through $\delta_4 > \delta$).

The response patterns in Figure 10 are overall consistent with responses to monetary policy shocks found by VAR studies (e.g., Christiano, Eichenbaum, and Evans [9]) and sought to match by sticky-price models (e.g., Christiano, Eichenbaum, and Evans [10]). Our model may mimic more closely the empirical patterns; for example, with a larger N and suitable χ_1 and ρ_t, the output response may move up more smoothly and peak before the inflation response peaks.

6 Discussion

Here we first discuss three settings in our model. The first setting is the lottery-purchasing scheme that endogenizes a limited participation in a money injection. Admittedly this scheme, as many modeling devices in economics, including the familiar helicopter drop, may not be directly observed in reality. In a world where the distribution of wealth is not degenerate, the scheme may be regarded as a parsimonious way to capture the dispersion of wealth induced by a money injection. On an abstract level, there is a simple rationale for the scheme: when some money is out, there ought to be some competition to get it; competition literally takes money from you and you are not guaranteed to win. From a realistic perspective, financial institutions may be the first recipients of money injected by the central bank. To benefit from the injection, you may need to be a shareholder of a financial institution; becoming a shareholder takes money and, again, you are not guaranteed to win. We suspect that an injection would induce a strong output response from such a financial-institution channel (of course, details of the channel must be carefully spelled out).

The second setting is our parameterization of preferences. We concentrate on a class of preferences that are widely used in recent monetary economics (see, e.g., Gali [16], Woodford [43], and Christiano, Eichenbaum, and Evans [10]). With such a preference, the steady-state output set takes the shape as in Figure 2, which is critical to our findings. This shape is quite robust when we vary underlying parameters (σ, η, F, and ω). Is it so under another class of preferences? Using a disutility function that is much more convex than the one in (1) (given η is bounded away from zero), Molico [30] presents a steady-state output set that is almost concave (the output curve in the left graph of Figure 1 is actually concave).[31] But after the two parameters in his disutility function are adjusted to make the function sufficiently flat, the shape of the output set turns into the one in Figure 2. More generally, we have a conjecture based on a feature of the model. The feature is that the transfers of goods in most pairwise meetings are around $y^* \equiv \arg\max_y [u(y) - c(y/A)]$ in a steady state. The conjecture is that the output set may take the shape in Figure 2 if $y \mapsto c(y/A)$ is not very convex over a sufficiently wide range, say, from $y^*/3$ to $3y^*$.

The third setting is the linear relationship $y = Al$ between output y in a meeting and the labor input l in the meeting. On the meeting level, the linear relationship seems plausible. For example, the driving distance that a taxi driver serves for a passenger is proportional to his driving time. The meeting-level linear relationship need not be inconsistent with a nonlinear relationship between aggregate output Y and the aggregate labor input L. If we assume $y = A\left(\frac{1}{L}\right)^{1-\alpha} l$, $\alpha \in (0,1]$, then $Y = AL^{\alpha}$; because this technology preserves the linear relationship between y and l, it has a rather minor effect on the output response. The externality imposed by L may be explained by some factor not explicitly modeled; in the taxi-driver example, the driving distance per hour may be affected by the number of occupied taxis on the road.

Next we note that Hume seemed to believe in the long-run stimulative effect of inflation:

> ...[I]t is of no manner of consequence, with regard to the domestic happiness of a state, whether money be in a greater or less quantity. The good policy of the magistrate consists only in keeping it, if possible, still increasing; because, by that means he keeps alive a spirit of industry in the nation. [Hume [20, p 173]]

It is straightforward to modify the basic model by making permanent the regressive money injection scheme in section 4 and study the steady-state output-inflation relationship. Now a higher inflation tends to reduce the value of money and hence output between every buyer-seller pair. But the regressive injection keeps the dispersing force on the distribution of money forever so that this effect may be very powerful—one may think of a one-shot injection when N is large and the injected

[31] Because the output set is largely concave, the logic of our reasoning in section 3 implies that a lump sum transfer of money should generate a positive output response. This is indeed the case. But the output response is insignificant because the lump sum transfer is not sufficiently progressive.

amount does not decline in these N periods. Offsetting this strong dispersing force by mixing the regressive injection with a repeated lump-sum money transfer, we find a positive output-inflation correlation when inflation is low and a negative correlation when inflation is high,[32] a pattern consistent with empirical findings for some economies including the U.S. (see, Ahmed and Rogers [1], Bullard [6], and Bullard and Keating [7]). Nonetheless, there is a constraint for a central bank to exploit this long-run relationship. A regressive injection always reduces average welfare in that $\sum v(m)\pi(m) > \sum v'(m)\pi'(m)$, where (v,π) is the zero-inflation steady state and (v',π') is the steady state with the stimulative injection. That is, less equality is a cost for higher output.

Finally we turn to two extensions that may be taken in the future. In our model, the monetary shock is one shot. While we do not see an obvious reason that our results would be overturned if monetary shocks are recurrent, it is certainly worth studying the extension with recurrent shocks. Apparently numerical analysis for this extension is much more challenging and may rely on some version of the method of Krusell and Smith [22]. For the second extension, we note that the output response in our model is essentially driven by people who are made poorer by an injection. There is no substantial role for people who are made richer by the injection. The latter group of people may have a much larger role if job creation is costly. The second extension therefore is to make the labor market distinguishable from the goods market and make it costly to create job in the labor market. Perceivably this is a demanding work, too.

[32] We report details in the online appendix.

Appendix A: Complete description of equilibria

A.1 The basic model

In section 2, $\lambda(m, m', v_{t+1})$ is defined as follows. For $d^+ \in \{1, ..., B-m\}$ and $d^- \in \{1, ...m\}$,

$$\lambda\left(m, m+d^+, v_{t+1}\right) = 0.5 \sum_{m^b=1}^{B} \pi_t\left(m^b\right) \mu\left(d^+; m^b, m, v_{t+1}\right), \tag{17}$$

$$\lambda\left(m, m-d^-, v_{t+1}\right) = 0.5 \sum_{m^s=0}^{B-1} \pi_t\left(m^s\right) \mu\left(d^-; m, m^s, v_{t+1}\right),$$

$$\lambda\left(m, m, v_{t+1}\right) = 0.5 \sum_{m^b=1}^{B} \pi_t\left(m^b\right) \mu\left(0; m^b, m, v_{t+1}\right) + 0.5 \sum_{m^s=0}^{B-1} \pi_t\left(m^s\right) \mu\left(0; m, m^s, v_{t+1}\right).$$

A.2 The model with centralized market

Consider the version of the model with a centralized market in the last part of section 3. The problem for a buyer with money holdings m is

$$\max_{y,\mu} \tilde{S}^b\left(y, \mu, m, \phi_t, v_{t+1}\right) \text{ s.t. } y = \phi_t \sum_d \mu(d)\, d, \tag{18}$$

where $\tilde{S}^b\left(y, \mu, m, \phi_t, v_{t+1}\right) = u(y) + \beta \sum \mu(d)\left[v_{t+1}(m-d) - v_{t+1}(m)\right]$; and the problem for a seller with m is

$$\max_{y,\mu} \tilde{S}^s\left(y, \mu, m, \phi_t, v_{t+1}\right) \text{ s.t. } y = \phi_t \sum_d \mu(d)\, d, \tag{19}$$

where $\tilde{S}^s\left(y, \mu, m, \phi_t, v_{t+1}\right) = -c(y/A) + \beta \sum \mu(d)\left[v_{t+1}(m+d) - v_{t+1}(m)\right]$. Let $(\tilde{y}^a(m, \phi_t, v_{t+1}), \tilde{\mu}^a(m, \phi_t, v_{t+1}))$ be the solution to the problem in (18) if $a = b$ and to the problem in (19) if $a = s$; then

$$v_t(m) = \beta v_{t+1}(m) + 0.5\left[\tilde{f}^b\left(m, \phi_t, v_{t+1}\right) + \tilde{f}^s\left(m, \phi_t, v_{t+1}\right)\right], \tag{20}$$

where $\tilde{f}^b$ and $\tilde{f}^s$, respectively, are the buyer's surplus and the seller's surplus implied by $(\tilde{y}^a, \tilde{\mu}^a)$. Also,

$$\pi_{t+1}(m) = \sum_{m'} \tilde{\lambda}\left(m', m, \phi_t, v_{t+1}\right) \pi_t\left(m'\right), \tag{21}$$

where $\tilde{\lambda}(m', m, \phi_t, v_{t+1})$ is the proportion of agents with m' units of money leaving with m after trading that is implied by $\left(\tilde{\mu}^b, \tilde{\mu}^s\right)$; that is, for $d^+ \in \{1, ..., B-m\}$ and $d^- \in \{1, ...m\}$,

$$\begin{aligned}
\tilde{\lambda}\left(m, m+d^+, \phi_t, v_{t+1}\right) &= 0.5\tilde{\mu}^b\left(d^+; m, \phi_t, v_{t+1}\right), \\
\tilde{\lambda}\left(m, m-d^-, \phi_t, v_{t+1}\right) &= 0.5\tilde{\mu}^s\left(d^-, m, \phi_t, v_{t+1}\right), \\
2\tilde{\lambda}\left(m, m, \phi_t, v_{t+1}\right) &= \tilde{\mu}^b\left(0; m, \phi_t, v_{t+1}\right) + \tilde{\mu}^s\left(0; m, \phi_t, v_{t+1}\right).
\end{aligned}$$

Market clearing requires

$$\sum_m \pi_t(m)\, \tilde{y}^s(m, \phi_t, v_{t+1}) = \sum_m \pi_t(m)\, \tilde{y}^b(m, \phi_t, v_{t+1}). \tag{22}$$

Given π_0, a sequence $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ is an equilibrium if it satisfies (20)-(22).

A.3 The basic model with one-shot injection

Turning to section 4, consider $t \in \{1, ..., N\}$. Let $\breve{\pi}_t$ be as given in the main text and let $f^b(.)$, $f^s(.)$, and $\lambda(.)$ be the same as in section 2 and A.1. Then we have

$$\pi_{t+1}(m) = \sum \lambda(m', m, v_{t+1})\, \breve{\pi}_t(m'), \tag{23}$$

and the value of holding m units of money right before pairwise meetings is

$$\breve{v}_t(m) = \beta v_{t+1}(m) + 0.5 \sum_{m'} \breve{\pi}_t(m')\left[f^b(m, m', v_{t+1}) + f^s(m', m, v_{t+1})\right]. \tag{24}$$

At the money injection stage, the problem for an agent with m units of money is

$$v_t(m) = \max_{x \le \min(m, B-m)} \chi_t \breve{v}_t(m+x) + (1-\chi_t)\, \breve{v}_t(m-x); \tag{25}$$

let $x(m, \chi_t, \breve{v}_t)$ be the solution to the problem in (25). Then

$$\breve{\pi}_t(m) = \sum_{m'} \check{\lambda}(m', m, \chi_t, \breve{v}_t)\, \pi_t(m'), \tag{26}$$

where $\check{\lambda}(m', m, \chi_t, \breve{v}_t)$ is the proportion of agents with m' units of money leaving money injection stage with m; that is, $\check{\lambda}(m, m+x, \chi_t, \breve{v}_t) = \chi_t$ and $\check{\lambda}(m, m-x, \chi_t, \breve{v}_t) = 1-\chi_t$ for all m with $x = x(m, \chi_t, \breve{v}_t) > 0$, and $\check{\lambda}(m, m, \chi_t, \breve{v}_t) = 1$ for all m with $x = x(m, \chi_t, \breve{v}_t) = 0$. Given $\{\chi_t\}_{t=1}^{N}$ and $\pi_1 = \pi$, a sequence $\{v_t, \pi_{t+1}\}_{t=1}^{\infty}$ together with $\{\breve{\pi}_t\}_{t=1}^{N}$ is an equilibrium if (23)-(26) hold for all $t \in \{1, ..., N\}$ and (3)-(8) hold for all $t > N$.

A.4 The model with bonds

For the model with bonds in section 5, let v_t, π_t, and $\hat{\pi}_t$ be as given in the main text. At period $t+1$, each unit of money disintegrates with probability $\delta\left(\hat{\pi}_t\right) = 1 - M/\left(\sum_\zeta \hat{\pi}_t\left(\zeta\right) w\right)$. Therefore the value of holding m units of nominal wealth at the start of $t+1$ is

$$\mathring{v}_{t+1}\left(m\right) = \sum_{m' \leq m} \binom{m}{m'} \left(1 - \delta\left(\hat{\pi}_t\right)\right)^{m'} \delta\left(\hat{\pi}_t\right)^{m-m'} v_{t+1}\left(m'\right). \tag{27}$$

At stage 2 of t, between a buyer with portfolio $\zeta^b = \left(m^b, w^b\right)$ and a seller with $\zeta^s = \left(m^s, w^s\right)$, the equilibrium meeting outcome $(y(\zeta^b, \zeta^s, \mathring{v}_{t+1}), \mu(\zeta^b, \zeta^s, \mathring{v}_{t+1}))$ is determined by (3), where we replace $\left(m^b, m^s, v_{t+1}\right)$ with $\left(\zeta^b, \zeta^s, \mathring{v}_{t+1}\right)$ and treat μ as a probability measure on $\left\{0, \ldots, \min\left(m^b, B - w^s\right)\right\}$. It follows that the value of holding portfolio $\xi \in \Xi$ before pairwise meetings is

$$\hat{v}_t\left(\zeta\right) = \beta \mathring{v}_{t+1}\left(w\right) + 0.5 \sum_{\zeta'} \hat{\pi}_t\left(\zeta'\right)\left[f^b\left(\zeta, \zeta', \mathring{v}_{t+1}\right) + f^s\left(\zeta', \zeta, \mathring{v}_{t+1}\right)\right], \tag{28}$$

where f^b and f^s are the buyer's surplus and the seller's surplus from the equilibrium meeting outcome, respectively, and $\lambda\left(\zeta', m, \mathring{v}_{t+1}\right)$ is the proportion of agents carrying portfolio ζ' into pairwise meetings and leaving with m units of nominal wealth (its description is similar to the one in (17) and skipped here). Also, we have

$$\pi_{t+1}\left(m\right) = \sum_{m' \geq m} \binom{m'}{m} \left(1 - \delta\left(\hat{\pi}_t\right)\right)^{m} \delta\left(\hat{\pi}_t\right)^{m'-m} \mathring{\pi}_{t+1}\left(m'\right), \tag{29}$$

where $\mathring{\pi}_{t+1}\left(m\right) = \sum_{\zeta'} \lambda\left(\zeta', m, \mathring{v}_{t+1}\right) \hat{\pi}_t\left(\zeta'\right)$ is the proportion of agents who hold m units of nominal wealth at the start of $t+1$. At the bond market of t, the problem for an agent with m units of money is

$$v_t\left(m\right) = \max_{\hat{\mu}} \sum_{\zeta'} \hat{\mu}(\zeta') \cdot \hat{v}_t\left(\zeta'\right) \tag{30}$$

subject to (14); let $\hat{\mu}\left(.; m, p_t^B, \hat{v}_t\right)$ be the solution to the problem in (30). Then the distribution $\hat{\pi}_t$ must satisfy

$$\hat{\pi}_t\left(\zeta\right) = \sum_{m'} \hat{\mu}\left(\zeta; m', p_t^B, \hat{v}_t\right) \pi_t\left(m'\right), \tag{31}$$

and clearing the bond market requires

$$\sum_{m}\left\{\pi_t\left(m\right)\left[\sum_{\zeta'=(m',w')}\left(w'-m'\right)\hat{\mu}\left(\zeta';m,p_t^B,\hat{v}_t\right)\right]\right\}=D_t. \tag{32}$$

Given π_0 and $\{D_t\}_{t\geq 0}$, a sequence $\{v_t,\hat{\pi}_t,\pi_{t+1},p_t^B\}_{t\geq 0}$ is an equilibrium if it satisfies (27)–(32). If there is a money injection, then we can introduce $\breve{\pi}_t$ as in A.3 for $t\in\{1,\ldots,N\}$.

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